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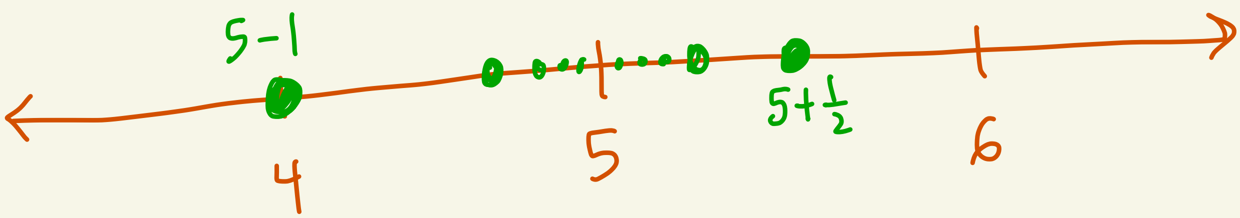
Test 1 Solutions

1 (a)

$$Y = \left\{ 5-1,\ 5+\tfrac{1}{2},\ 5-\tfrac{1}{3},\ 5+\tfrac{1}{4},\ 5-\tfrac{1}{5},\ \dots \right\}$$



$\inf(Y) = 4$

$\sup(Y) = 5.5$

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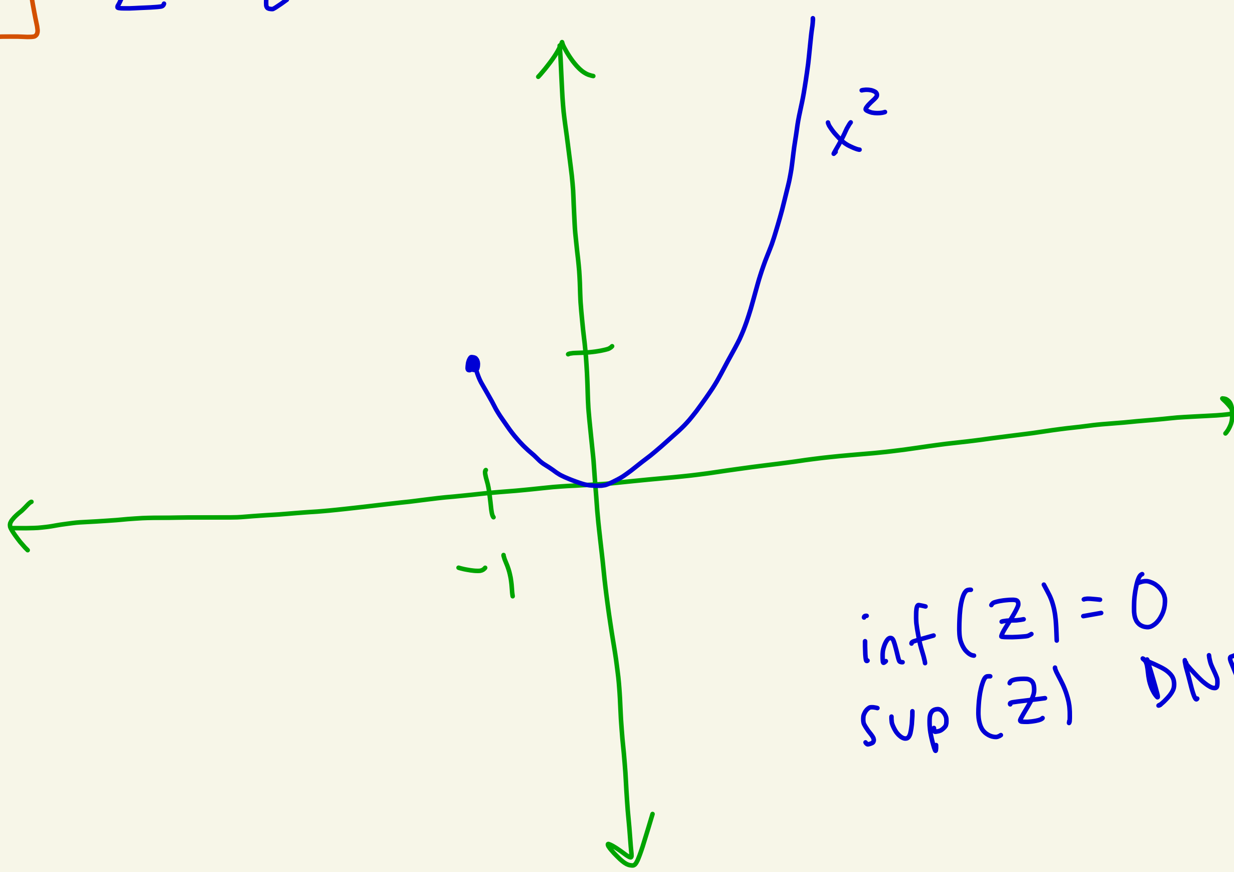
1 (b)

$$Z = \{ x^2 \mid x \in \mathbb{R} \text{ with } -1 \leq x \}$$



$\inf(Z) = 0$

$\sup(Z)$ DNE

(2) Let $\varepsilon > 0$.

Note that

$$\left| \frac{n}{2n+1} - \frac{1}{2} \right| = \left| \frac{2n - 2n - 1}{2(2n+1)} \right|$$

$$= \left| \frac{-1}{4n+2} \right| = \frac{1}{4n+2}$$

We have that $\frac{1}{4n+2} < \varepsilon$

iff $\frac{1}{\varepsilon} < 4n+2$

iff $\frac{1}{\varepsilon} - 2 < 4n$

iff $\frac{1}{4\varepsilon} - \frac{1}{2} < n$

Let $N > \frac{1}{4\varepsilon} - \frac{1}{2}$

Then if $n \geq N > \frac{1}{4\varepsilon} - \frac{1}{2}$ we have that

$$\left| \frac{n}{2n+1} - \frac{1}{2} \right| = \frac{1}{2n+1} < \varepsilon.$$

Thus, $\lim_{n \to \infty} \frac{n}{2n+1} = \frac{1}{2}$

(A) HW 1 # 6(a)

(B) HW 1 # 7(a)

(C) HW 1 # 8(b)

(D) See class notes from 9/8 pg. 6.

(E) HW 2 #4(b)

(F) HW 2 # 5
